EXAMPLE 6.4: A transform with complex-valued basis vectors.

Unlike orthogonal transformation matrices, where the inverse of the transformation matrix is its transpose, the inverse of unitary transformation matrix

$$\mathbf{A} = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 - j0.866 & -0.5 + j0.866 \\ 1 & -0.5 + j0.866 & -0.5 - j0.866 \end{bmatrix} \tag{6-45}$$

is its conjugate transpose. Thus,

$$\begin{aligned} \mathbf{A}^{*T}\mathbf{A} &= \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 - j0.866 & -0.5 + j0.866 \\ 1 & -0.5 + j0.866 & -0.5 - j0.866 \end{bmatrix}^{*T} \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 - j0.866 & -0.5 + j0.866 \\ 1 & -0.5 + j0.866 & -0.5 - j0.866 \end{bmatrix} \\ &= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 + j0.866 & -0.5 - j0.866 \\ 1 & -0.5 - j0.866 & -0.5 + j0.866 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & -0.5 - j0.866 & -0.5 + j0.866 \\ 1 & -0.5 + j0.866 & -0.5 - j0.866 \end{bmatrix} \\ &= \frac{1}{3} \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = \mathbf{I} \end{aligned}$$

where $j = \sqrt{-1}$ and matrix **A** is a unitary matrix that can be used in **Eqs. (6-41)** through **(6-44)**. It is easy to show (see **Problem 6.4**) that when $\mathbf{A}^{*T}\mathbf{A} = \mathbf{I}$, the basis vectors in **A** satisfy **Eq. (6-40)** and are thus orthonormal.